EE-565 – W7
INDUCTION MACHINE MODELING

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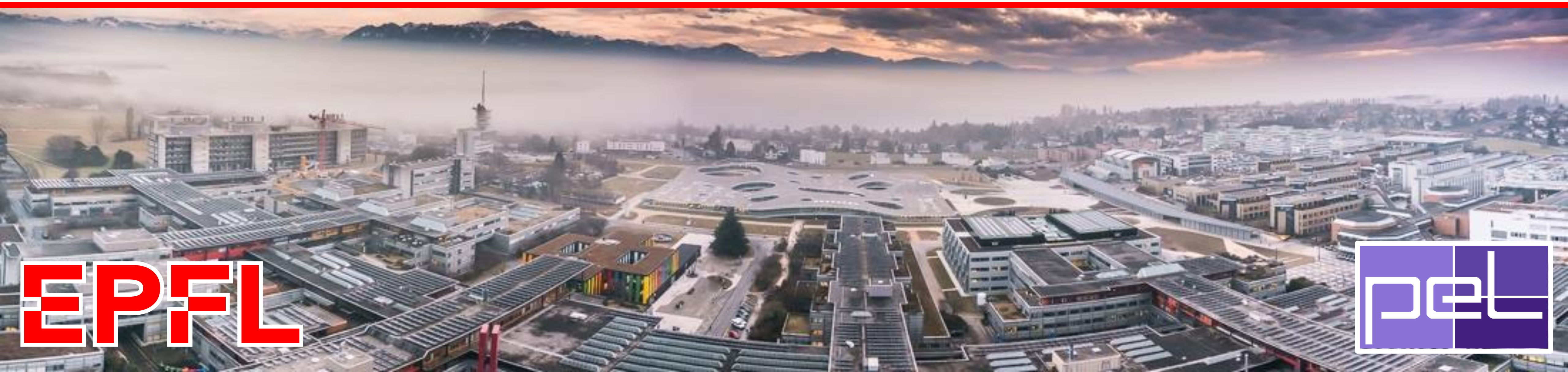


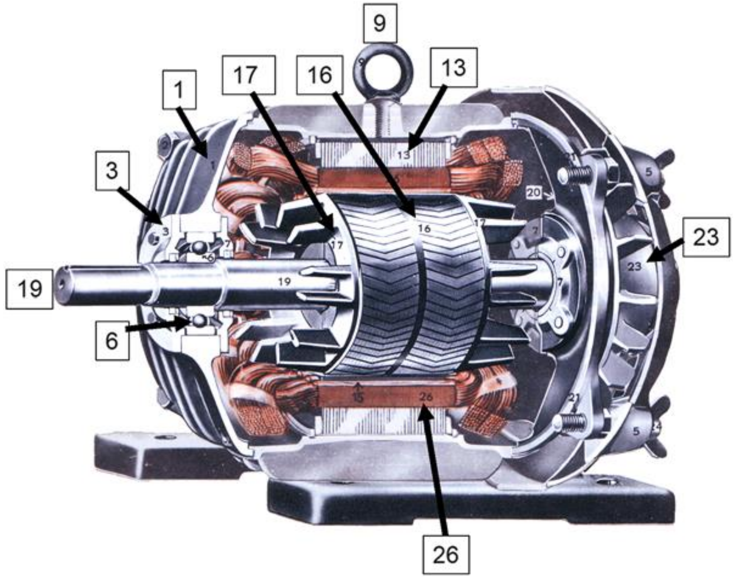
INDUCTION MACHINE FUNDAMENTALS

Operating principle, construction details

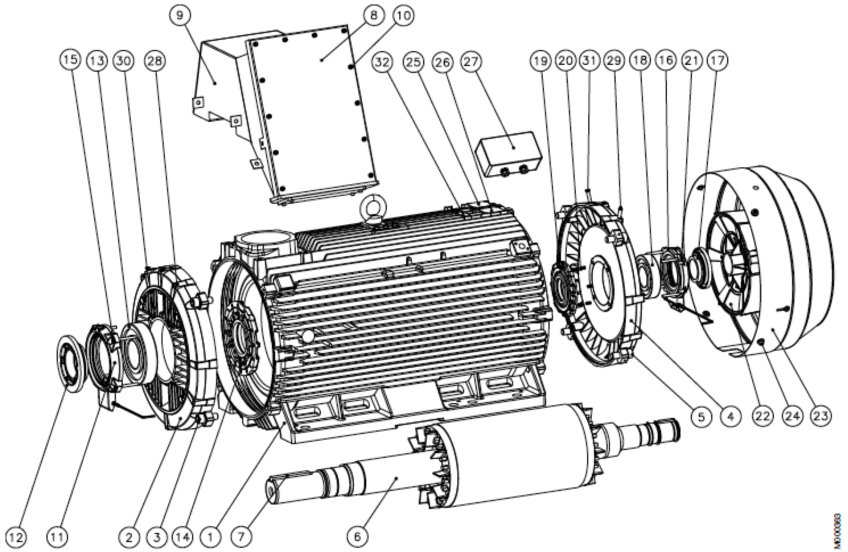
INDUCTION MACHINES

Induction machines are the “**Workhorse**” of modern industry

- ► simple, reliable and robust design
- ► low maintenance effort
- ► stator (winding arranged in slots)
- ► rotor (squirell cage, shorted winding)



Typical exploded view of high voltage cast iron motor

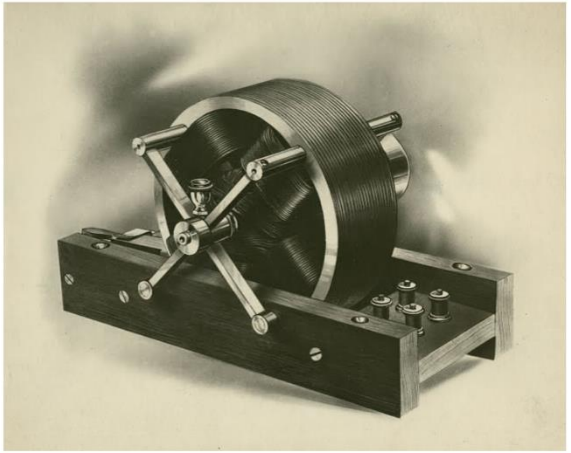


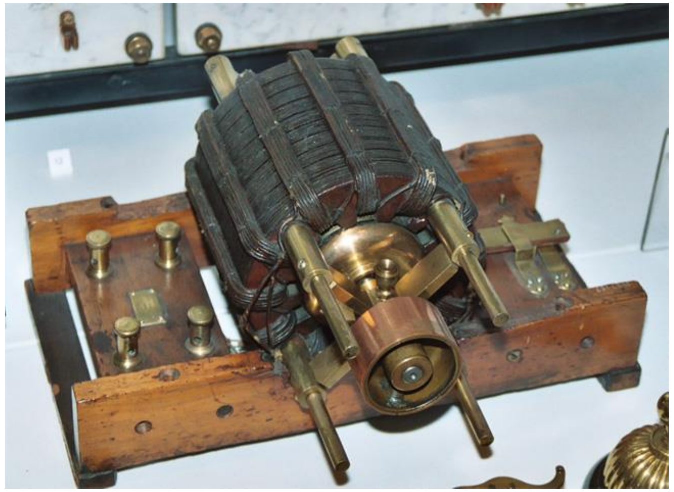
1 Stator frame
2 End shield, D-end
3 Screws for end shield, D-end
4 End shield, N-end
5 Screws for end shield, N-end
6 Rotor with shaft
7 Key, D-end
8 Main terminal box
9 Middle box
10 Screws for terminal box cover
11 Outer bearing cover, D-end
12 Valve disc with labyrinth seal, D-end
13 Bearing, D-end
14 Inner bearing cover
15 Screws for bearing cover, D-end
16 Outer bearing cover, N-end
17 Valve disc with labyrinth seal, D-end
18 Bearing, N-end
19 Inner bearing cover, N-end
20 Spring
21 Screws for bearing cover, N-end
22 Fan
23 Fan cover
24 Screws for fan cover
25 Rating plate
26 Lubrication plate
27 Auxiliary terminal box
28 Grease nipple, D-end
29 Grease nipple, N-end
30 SPM nipple, D-end
31 SPM nipple, N-end
32 Additional identification plate

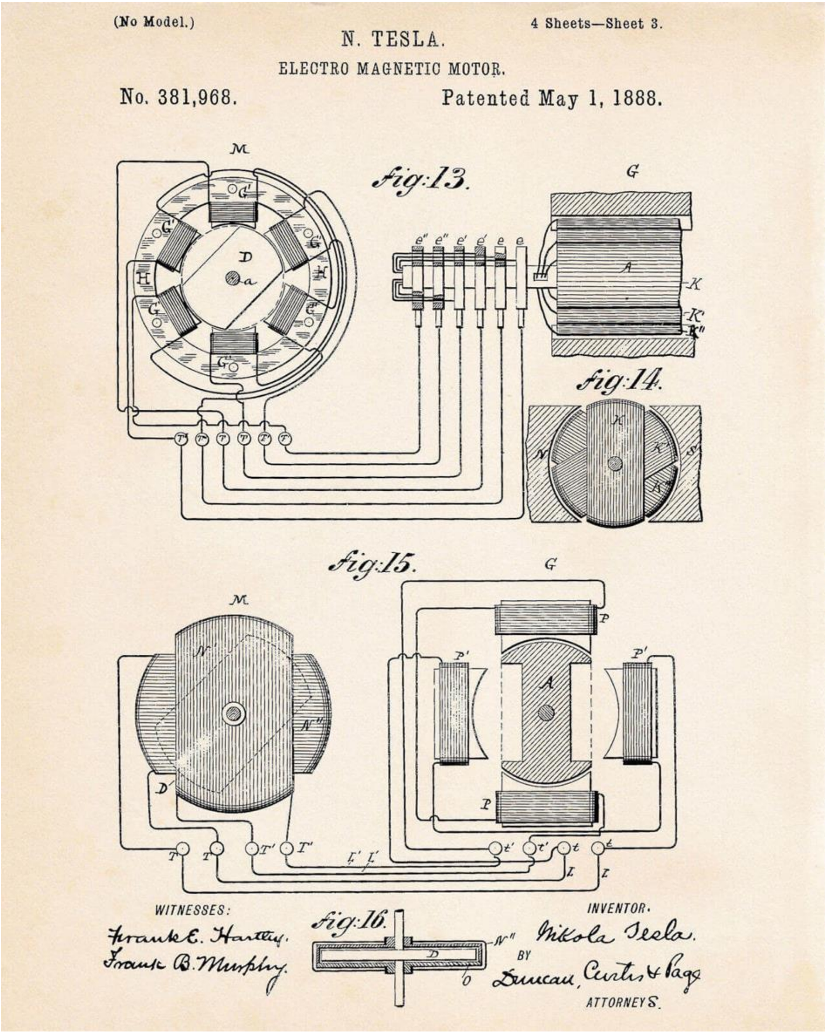
HISTORY

More than 100 years of use

- 1885-1887 – First demonstrators (N.Tesla, G.Ferraris)
- 1887-1888 – Patens
- 1889 – Squirrel Cage rotor
- 1890 – First commercial products (GE, Westinghouse)





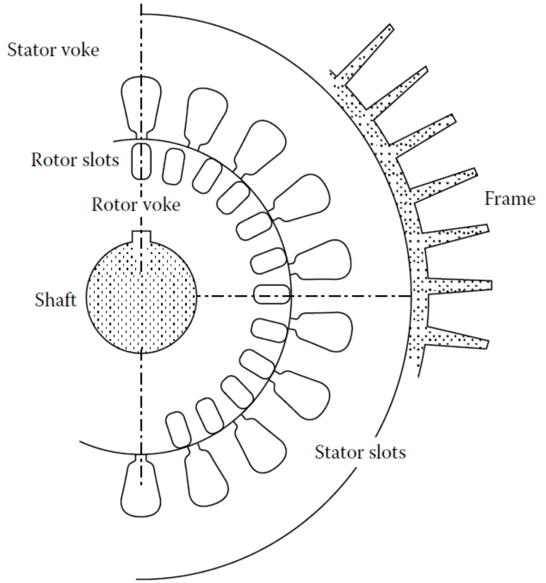


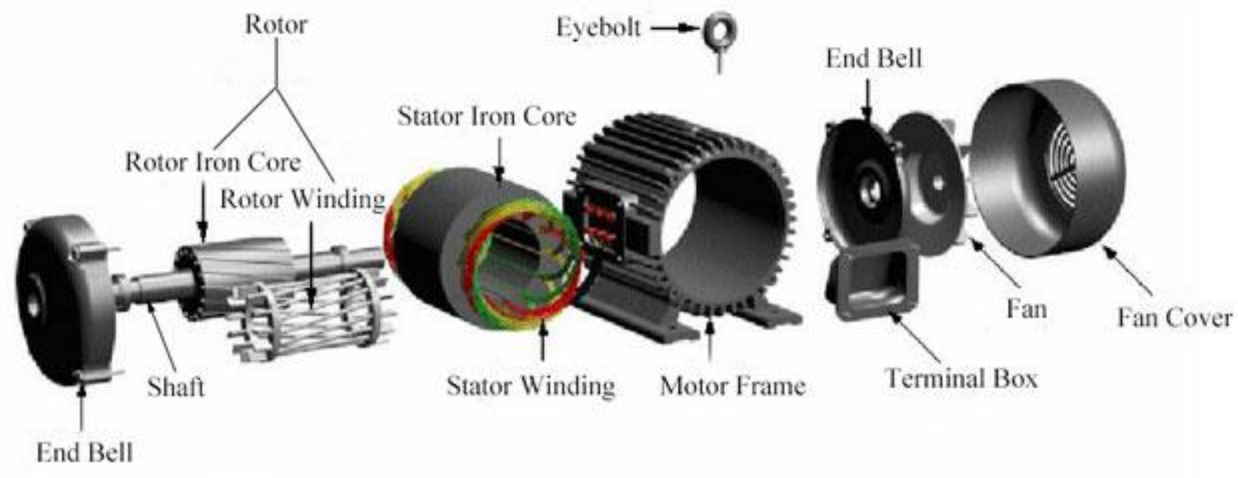
CONSTRUCTION AND TYPE OF MACHINES

Induction machines consists of

- **Stator**
- **Rotor**
  - **Squirrel Cage Rotor**
  - **Wound Rotor (slip ring)**

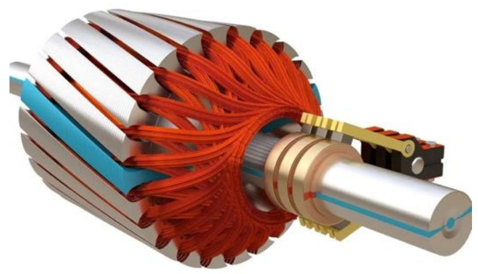
Different IM types are classified based on the rotor





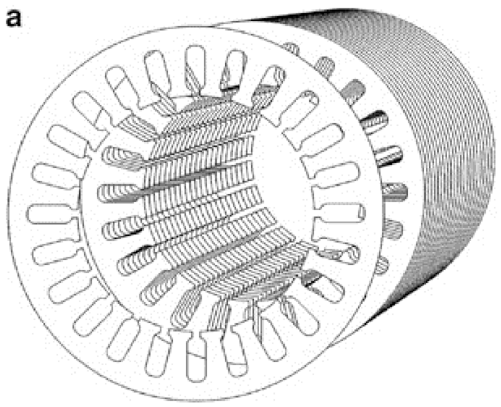
Squirrel Cage

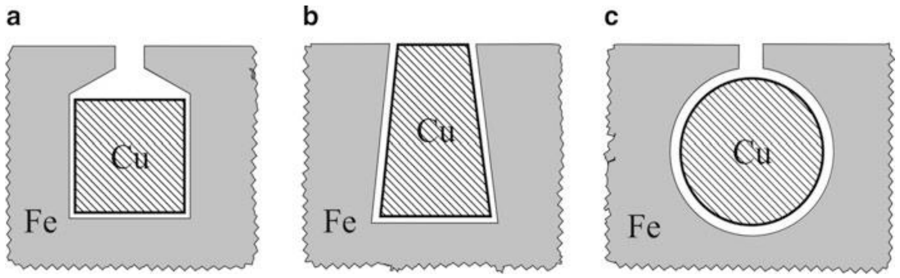
Wound Rotor

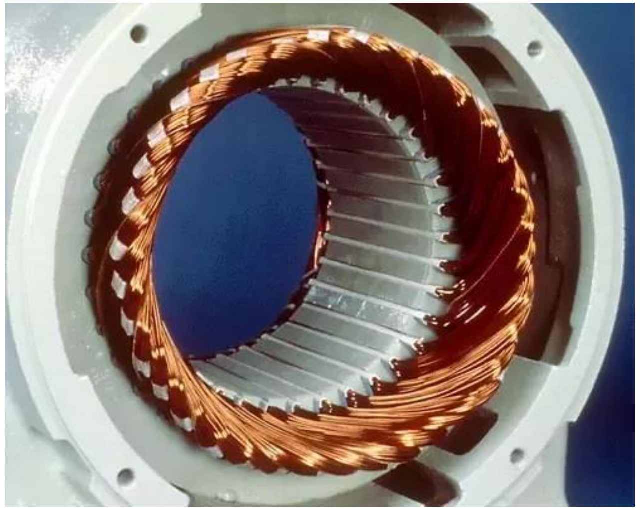


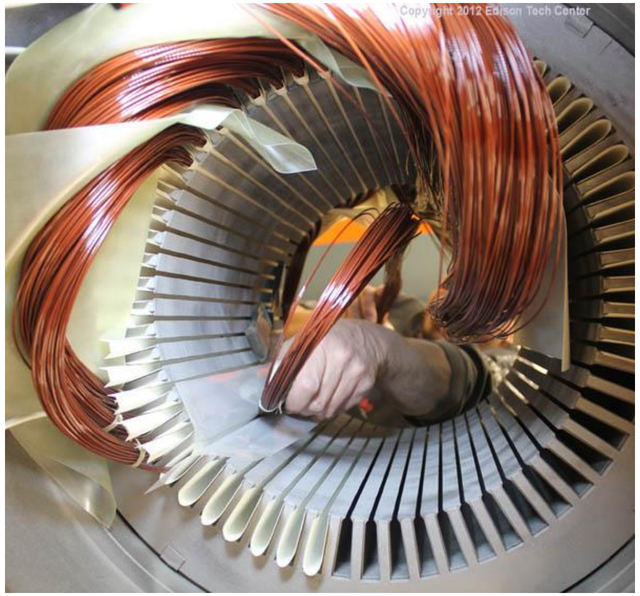
STATOR

Stator windings are subject to AC frequency current
Stator core is realized with **laminated sheets** to reduce eddy current losses
Different slot shapes are used





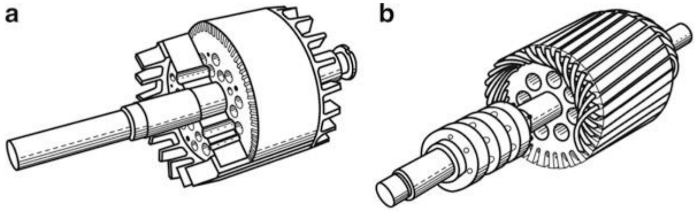


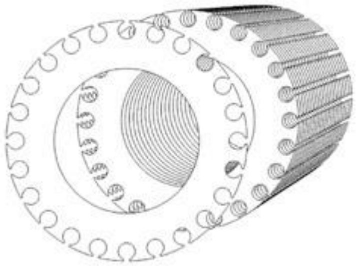


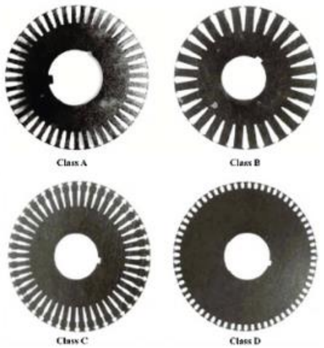
ROTOR

Rotor windings are subject to AC frequency current (typically low frequency)
Rotor core is also (usually) realized with **laminated sheets** to reduce eddy current losses

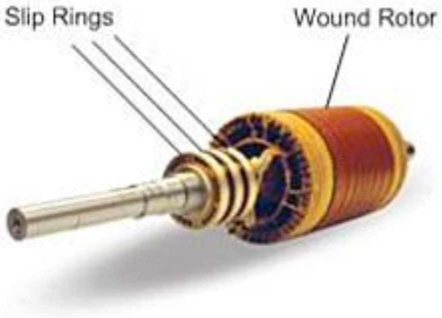
The rotor can be classified in **Wound Rotor** and **Squirrel Cage Rotor**



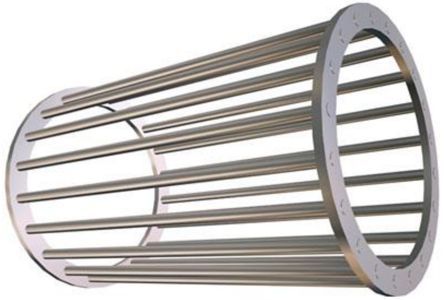




Wound rotor is realized with AC windings located in the slots
In this case, typically the rotor winding terminals are accessible via **slip rings**
In the past, this was mostly used for speed regulation (rotor impedance)
Nowadays it is mostly used in **Doubly Fed Induction Machines (DFIMs)**



Squirrel cage rotor is realized with **solid bars**
The bars are short circuited at the extremes **(end rings)**
The terminals are not externally accessible (no mechanical contact)
The bars are often tilted (**Skewing**) to reduce cogging torque effects



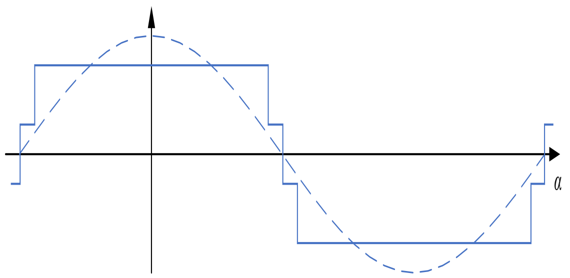
INDUCTION MACHINE PHASE VARIABLE MODELING

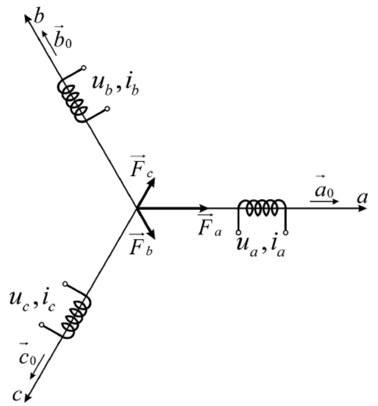
Modeling in the natural reference frame

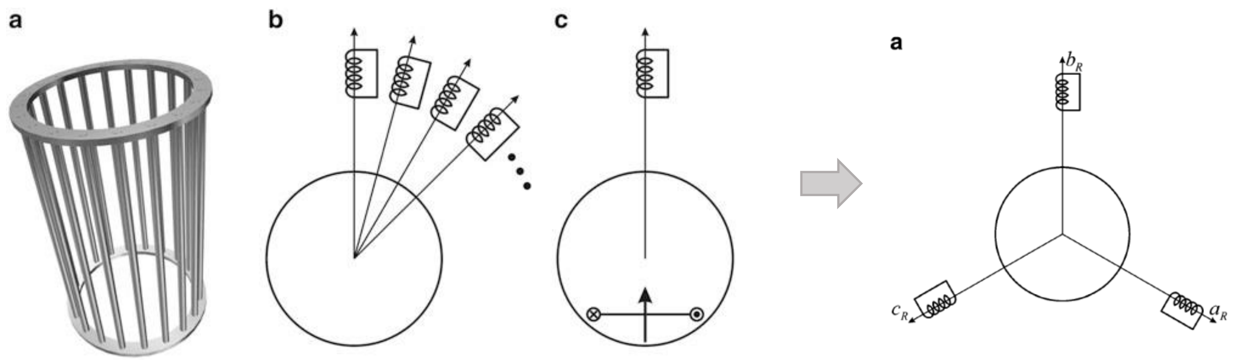
SIMPLIFYING ASSUMPTIONS

The mathematical model of the Induction machine is derived under some simplifying assumptions

- The **air-gap MMF is perfectly sinusoidal** (no higher order spatial harmonics)
- The **stator** has a **symmetrical three-phase winding** (same number of turns, same arrangement, 120° phase shift)
- The **rotor** is modeled as a **symmetrical three-phase winding** (same number of turns, same arrangement, 120° phase shift)
  - Exact model for wound rotors, simplified model for squirrel cage rotor
- The stator and the rotor windings have the **same number of pole pairs**
- The **rotor** is perfectly **short-circuited** (no external impedance or external rotor source is considered)
- Hysteresis and eddy current losses in the iron are neglected (or added semi-empirically in a later stage)
- Cogging torque is neglected







INDUCTION MACHINE MODEL – FROM THE GENERAL MODEL

$$\boldsymbol{v} = \begin{bmatrix} \boldsymbol{v}_s \\ \boldsymbol{v}_r \end{bmatrix} = \begin{bmatrix} v_{s,a} \\ v_{s,b} \\ v_{s,c} \\ v_{r,a} \\ v_{r,b} \\ v_{r,c} \end{bmatrix}$$

- $v_{s,a}, v_{s,b}, v_{s,c}$: Stator Voltages
- $v_{r,a}, v_{r,b}, v_{r,c}$: Rotor Voltages (=0)

$$\boldsymbol{i} = \begin{bmatrix} \boldsymbol{i}_s \\ \boldsymbol{i}_r \end{bmatrix} = \begin{bmatrix} i_{s,a} \\ i_{s,b} \\ i_{s,c} \\ i_{r,a} \\ i_{r,b} \\ i_{r,c} \end{bmatrix}$$

- $i_{s,a}, i_{s,b}, i_{s,c}$: Stator Currents
- $i_{r,a}, i_{r,b}, i_{r,c}$: Rotor Currents

$$\boldsymbol{\phi} = \begin{bmatrix} \boldsymbol{\phi}_s \\ \boldsymbol{\phi}_r \end{bmatrix} = \begin{bmatrix} \phi_{s,a} \\ \phi_{s,b} \\ \phi_{s,c} \\ \phi_{r,a} \\ \phi_{r,b} \\ \phi_{r,c} \end{bmatrix}$$

- $\phi_{s,a}, \phi_{s,b}, \phi_{s,c}$: Stator Fluxes
- $\phi_{r,a}, \phi_{r,b}, \phi_{r,c}$: Rotor Fluxes

Electrical Equations

$$\begin{cases} \boldsymbol{v} = \boldsymbol{R} \cdot \boldsymbol{i} + \dfrac{\mathrm{d}\boldsymbol{\phi}}{\mathrm{d}t} \\ \boldsymbol{\phi} = \cancel{\boldsymbol{\psi}_{PM}(\theta_m)} + \boldsymbol{L}(\theta_m) \cdot \boldsymbol{i} \end{cases}$$

Electrical Angle Definition

$$\theta_e = P_p \theta_m$$

Mechanical Equations

$$\begin{cases} \dfrac{\mathrm{d}\theta_m}{\mathrm{d}t} = \omega_m \\ J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{d}t} + F(\omega_m) \cdot \omega_m = T_{em} - T_m \\ T_{em} = \cancel{\dfrac{\partial W'^{(0)}_{em}}{\partial \theta_m}} + \cancel{\dfrac{\partial \boldsymbol{\psi}^{\mathrm{T}}_{PM}}{\partial \theta_m}} \cdot \boldsymbol{i} + \dfrac{1}{2} \boldsymbol{i}^{\mathrm{T}} \cdot \dfrac{\partial \boldsymbol{L}}{\partial \theta_m} \cdot \boldsymbol{i} \end{cases}$$

INDUCTION MACHINE MODEL – VOLTAGE BALANCE EQUATIONS

$$\boldsymbol{v} = \boldsymbol{R} \cdot \boldsymbol{i} + \frac{\mathrm{d}\boldsymbol{\phi}}{\mathrm{d}t}$$

⇓

$$\begin{bmatrix} v_{s,a} \\ v_{s,b} \\ v_{s,c} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} v_{s,a} \\ v_{s,b} \\ v_{s,c} \\ v_{r,a} \\ v_{r,b} \\ v_{r,c} \end{bmatrix} = \begin{bmatrix} R_s & 0 & 0 & 0 & 0 & 0 \\ 0 & R_s & 0 & 0 & 0 & 0 \\ 0 & 0 & R_s & 0 & 0 & 0 \\ 0 & 0 & 0 & R_r & 0 & 0 \\ 0 & 0 & 0 & 0 & R_r & 0 \\ 0 & 0 & 0 & 0 & 0 & R_r \end{bmatrix} \cdot \begin{bmatrix} i_{s,a} \\ i_{s,b} \\ i_{s,c} \\ i_{r,a} \\ i_{r,b} \\ i_{r,c} \end{bmatrix} + \frac{\mathrm{d}}{\mathrm{d}t} \begin{bmatrix} \phi_{s,a} \\ \phi_{s,b} \\ \phi_{s,c} \\ \phi_{r,a} \\ \phi_{r,b} \\ \phi_{r,c} \end{bmatrix}$$

⇓

$$\begin{cases} \boldsymbol{v}_s = R_s \cdot \boldsymbol{i}_s + \dfrac{\mathrm{d}\boldsymbol{\phi}_s}{\mathrm{d}t} \\ \boldsymbol{0} = \boldsymbol{v}_r = R_r \cdot \boldsymbol{i}_r + \dfrac{\mathrm{d}\boldsymbol{\phi}_r}{\mathrm{d}t} \end{cases}$$

The voltage balance equations can be grouped in stator equations and rotor equations

INDUCTION MACHINE MODEL – FLUX EQUATIONS

$$\boldsymbol{\phi} = \boldsymbol{L} \cdot \boldsymbol{i} = \boldsymbol{L}_l \cdot \boldsymbol{i} + \boldsymbol{L}_m \cdot \boldsymbol{i}$$

The inductance matrix can be split into a **leakage inductance matrix contribution** and a **mutual inductance matrix contribution**

$$\boldsymbol{L}_l = \begin{bmatrix} L_{l,s} & 0 & 0 & 0 & 0 & 0 \\ 0 & L_{l,s} & 0 & 0 & 0 & 0 \\ 0 & 0 & L_{l,s} & 0 & 0 & 0 \\ 0 & 0 & 0 & L_{l,r} & 0 & 0 \\ 0 & 0 & 0 & 0 & L_{l,r} & 0 \\ 0 & 0 & 0 & 0 & 0 & L_{l,r} \end{bmatrix}$$

The leakage contribution considers fluxes that do not go in the air-gap.
All the windings are decoupled.

The mutual contribution considers fluxes that go in the air-gap.
All the windings are coupled.

$$\boldsymbol{L}_m = \begin{bmatrix} \boldsymbol{L}_{s,s} & \boldsymbol{L}_{s,r} \\ \boldsymbol{L}_{r,s} & \boldsymbol{L}_{r,r} \end{bmatrix} = \begin{bmatrix} L_{(s,a),(s,a)} & L_{(s,a),(s,b)} & L_{(s,a),(s,c)} & L_{(s,a),(r,a)} & L_{(s,a),(r,b)} & L_{(s,a),(r,c)} \\ L_{(s,b),(s,a)} & L_{(s,b),(s,b)} & L_{(s,b),(s,c)} & L_{(s,b),(r,a)} & L_{(s,b),(r,b)} & L_{(s,b),(r,c)} \\ L_{(s,c),(s,a)} & L_{(s,c),(s,b)} & L_{(s,c),(s,c)} & L_{(s,c),(r,a)} & L_{(s,c),(r,b)} & L_{(s,c),(r,c)} \\ L_{(r,a),(s,a)} & L_{(r,a),(s,b)} & L_{(r,a),(s,c)} & L_{(r,a),(r,a)} & L_{(r,a),(r,b)} & L_{(r,a),(r,c)} \\ L_{(r,b),(s,a)} & L_{(r,b),(s,b)} & L_{(r,b),(s,c)} & L_{(r,b),(r,a)} & L_{(r,b),(r,b)} & L_{(r,b),(r,c)} \\ L_{(r,c),(s,a)} & L_{(r,c),(s,b)} & L_{(r,c),(s,c)} & L_{(r,c),(r,a)} & L_{(r,c),(r,b)} & L_{(r,c),(r,c)} \end{bmatrix}$$

INDUCTION MACHINE MODEL – INDUCTANCES

The coefficients of the inductance matrix can be grouped depending on the influence stator-stator, rotor-rotor and stator-rotor

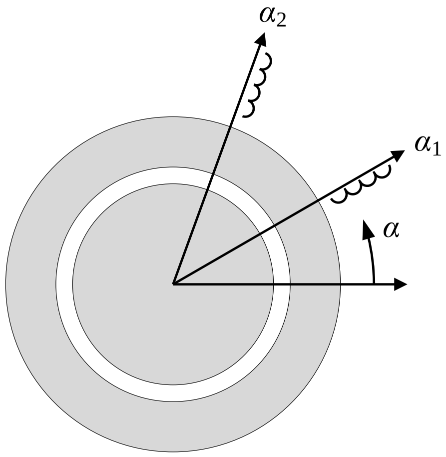
$$\boldsymbol{L}_m = \begin{bmatrix} \boldsymbol{L}_{s,s} & \boldsymbol{L}_{s,r} \\ \boldsymbol{L}_{r,s} & \boldsymbol{L}_{r,r} \end{bmatrix} = \begin{bmatrix} L_{(s,a),(s,a)} & L_{(s,a),(s,b)} & L_{(s,a),(s,c)} & L_{(s,a),(r,a)} & L_{(s,a),(r,b)} & L_{(s,a),(r,c)} \\ L_{(s,b),(s,a)} & L_{(s,b),(s,b)} & L_{(s,b),(s,c)} & L_{(s,b),(r,a)} & L_{(s,b),(r,b)} & L_{(s,b),(r,c)} \\ L_{(s,c),(s,a)} & L_{(s,c),(s,b)} & L_{(s,c),(s,c)} & L_{(s,c),(r,a)} & L_{(s,c),(r,b)} & L_{(s,c),(r,c)} \\ L_{(r,a),(s,a)} & L_{(r,a),(s,b)} & L_{(r,a),(s,c)} & L_{(r,a),(r,a)} & L_{(r,a),(r,b)} & L_{(r,a),(r,c)} \\ L_{(r,b),(s,a)} & L_{(r,b),(s,b)} & L_{(r,b),(s,c)} & L_{(r,b),(r,a)} & L_{(r,b),(r,b)} & L_{(r,b),(r,c)} \\ L_{(r,c),(s,a)} & L_{(r,c),(s,b)} & L_{(r,c),(s,c)} & L_{(r,c),(r,a)} & L_{(r,c),(r,b)} & L_{(r,c),(r,c)} \end{bmatrix}$$

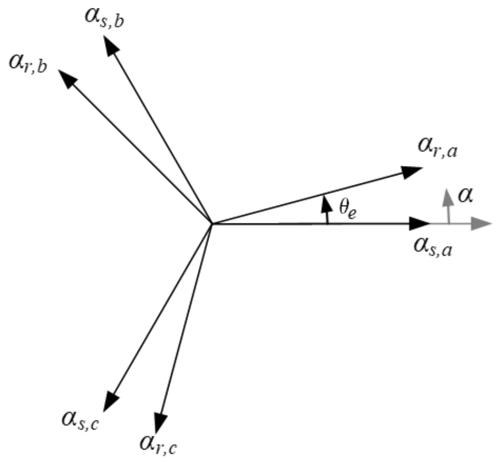
In general, the coefficients of the inductance matrix depend on the equivalent number of turns and on the phase shift between the magnetic axes of the windings

$$L_m = \frac{N_1' \, N_2'}{R_{m,eq}} \cos(\alpha_1 - \alpha_2)$$

The magnetic axes are assigned by the geometry

$$\alpha_{s,a} = 0^\circ \qquad \alpha_{r,a} = \theta_e$$
$$\alpha_{s,b} = 120^\circ \qquad \alpha_{s,a} = \theta_e + 120^\circ$$
$$\alpha_{s,c} = 240^\circ \qquad \alpha_{s,a} = \theta_e + 240^\circ$$





INDUCTION MACHINE MODEL – INDUCTANCES

The stator-stator inductances do not depend on the relative position of the rotor

$$\boldsymbol{L}_{s,s} = \frac{N_s'^2}{R_{m,eq}} \begin{bmatrix} \cos(\alpha_{s,a}-\alpha_{s,a}) & \cos(\alpha_{s,a}-\alpha_{s,b}) & \cos(\alpha_{s,a}-\alpha_{s,c}) \\ \cos(\alpha_{s,b}-\alpha_{s,a}) & \cos(\alpha_{s,b}-\alpha_{s,b}) & \cos(\alpha_{s,b}-\alpha_{s,c}) \\ \cos(\alpha_{s,c}-\alpha_{s,a}) & \cos(\alpha_{s,c}-\alpha_{s,b}) & \cos(\alpha_{s,c}-\alpha_{s,c}) \end{bmatrix} = L_{m,s} \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}$$

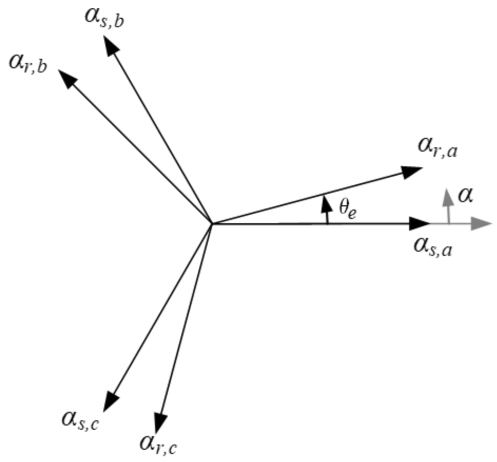
The rotor-rotor interactions do not depend on the relative position of the rotor

$$\boldsymbol{L}_{r,r} = \frac{N_r'^2}{R_{m,eq}} \begin{bmatrix} \cos(\alpha_{r,a}-\alpha_{r,a}) & \cos(\alpha_{r,a}-\alpha_{r,b}) & \cos(\alpha_{r,a}-\alpha_{r,c}) \\ \cos(\alpha_{r,b}-\alpha_{r,a}) & \cos(\alpha_{r,b}-\alpha_{r,b}) & \cos(\alpha_{r,b}-\alpha_{r,c}) \\ \cos(\alpha_{r,c}-\alpha_{r,a}) & \cos(\alpha_{r,c}-\alpha_{r,b}) & \cos(\alpha_{r,c}-\alpha_{r,c}) \end{bmatrix} = L_{m,r} \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix}$$

Only the stator-rotor interactions depend on the relative position of the rotor

$$\boldsymbol{L}_{s,r} = \frac{N_s' N_r'}{R_{m,eq}} \begin{bmatrix} \cos(\alpha_{s,a}-\alpha_{r,a}) & \cos(\alpha_{s,a}-\alpha_{r,b}) & \cos(\alpha_{s,a}-\alpha_{r,c}) \\ \cos(\alpha_{s,b}-\alpha_{r,a}) & \cos(\alpha_{s,b}-\alpha_{r,b}) & \cos(\alpha_{s,b}-\alpha_{r,c}) \\ \cos(\alpha_{s,c}-\alpha_{r,a}) & \cos(\alpha_{s,c}-\alpha_{r,b}) & \cos(\alpha_{s,c}-\alpha_{r,c}) \end{bmatrix} =$$

$$= L_{m,sr} \begin{bmatrix} \cos(\theta_e) & \cos(\theta_e + 120°) & \cos(\theta_e + 240°) \\ \cos(\theta_e - 120°) & \cos(\theta_e) & \cos(\theta_e + 120°) \\ \cos(\theta_e - 240°) & \cos(\theta_e - 120°) & \cos(\theta_e) \end{bmatrix}$$

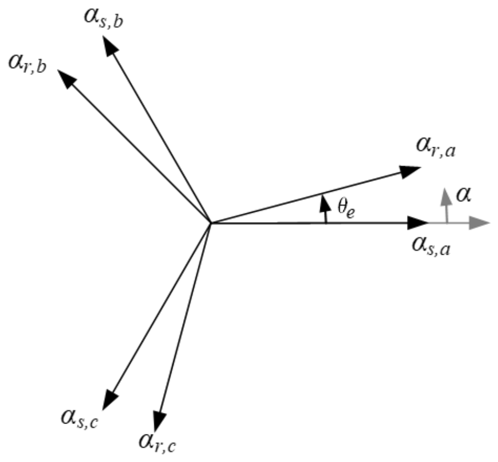


INDUCTION MACHINE MODEL – INDUCTANCES

Based on the previous results

$$\boldsymbol{\phi}_s = L_{l,s} \cdot \boldsymbol{i}_s + \boldsymbol{L}_{s,s} \cdot \boldsymbol{i}_s + \boldsymbol{L}_{s,r}(\theta_e) \cdot \boldsymbol{i}_r$$

$$\boldsymbol{\phi}_r = L_{l,r} \cdot \boldsymbol{i}_r + \boldsymbol{L}_{r,r} \cdot \boldsymbol{i}_r + \boldsymbol{L}_{s,r}^{\mathrm{T}}(\theta_e) \cdot \boldsymbol{i}_s$$



With the inductances expressed as:

$$\boldsymbol{L}_{s,s} = L_{m,s} \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix} \qquad L_{m,s} = \frac{{N'_s}^2}{R_{m,eq}}$$

$$\boldsymbol{L}_{r,r} = L_{m,r} \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix} \qquad L_{m,r} = \frac{{N'_r}^2}{R_{m,eq}}$$

$$\boldsymbol{L}_{s,r} = L_{m,sr} \begin{bmatrix} \cos(\theta_e) & \cos(\theta_e + 120°) & \cos(\theta_e + 240°) \\ \cos(\theta_e - 120°) & \cos(\theta_e) & \cos(\theta_e + 120°) \\ \cos(\theta_e - 240°) & \cos(\theta_e - 120°) & \cos(\theta_e) \end{bmatrix} \qquad L_{m,sr} = \frac{N'_s N'_r}{R_{m,eq}}$$

INDUCTION MACHINE MODEL – ELECTROMAGNETIC TORQUE

$$T_{em} = \frac{1}{2}\boldsymbol{i}^{\mathrm{T}} \cdot \frac{\partial \boldsymbol{L}}{\partial \theta_m} \cdot \boldsymbol{i}$$

We can express the torque in terms of the electrical angle

$$= P_p \cdot \frac{1}{2}\boldsymbol{i}^{\mathrm{T}} \cdot \frac{\partial \boldsymbol{L}}{\partial \theta_e} \cdot \boldsymbol{i}$$

We can split the stator and rotor contributions

$$= P_p \cdot \frac{1}{2}\begin{bmatrix}\boldsymbol{i}_s & \boldsymbol{i}_r\end{bmatrix} \cdot \frac{\partial}{\partial \theta_e}\begin{bmatrix}\boldsymbol{L}_{s,s} & \boldsymbol{L}_{s,r}(\theta_e) \\ \boldsymbol{L}_{s,r}^{\mathrm{T}}(\theta_e) & \boldsymbol{L}_{r,r}\end{bmatrix} \cdot \begin{bmatrix}\boldsymbol{i}_s \\ \boldsymbol{i}_r\end{bmatrix}$$

Only the stator-rotor interaction depends on the rotor position, and it is counted twice

$$= P_p \cdot \boldsymbol{i}_s^{\mathrm{T}} \cdot \frac{\partial \boldsymbol{L}_{s,r}}{\partial \theta_e} \cdot \boldsymbol{i}_r$$

The torque is generated by interaction between stator currents and rotor currents, through the matrix

$$\frac{\partial \boldsymbol{L}_{s,r}}{\partial \theta_e} = -L_{m,sr}\begin{bmatrix}\sin(\theta_e) & \sin(\theta_e + 120°) & \sin(\theta_e + 240°) \\ \sin(\theta_e - 120°) & \sin(\theta_e) & \sin(\theta_e + 120°) \\ \sin(\theta_e - 240°) & \sin(\theta_e - 120°) & \sin(\theta_e)\end{bmatrix}$$

INDUCTION MACHINE MODEL – PHASE VARIABLE DOMAIN

Electrical Equations

$$\begin{cases} \boldsymbol{v}_s = R_s \cdot \boldsymbol{i}_s + \dfrac{\mathrm{d}\boldsymbol{\phi}_s}{\mathrm{d}t} \\ \boldsymbol{0} = \boldsymbol{v}_r = R_r \cdot \boldsymbol{i}_r + \dfrac{\mathrm{d}\boldsymbol{\phi}_r}{\mathrm{d}t} \\ \boldsymbol{\phi}_s = L_{l,s} \cdot \boldsymbol{i}_s + \boldsymbol{L}_{s,s} \cdot \boldsymbol{i}_s + \boldsymbol{L}_{s,r}(\theta_e) \cdot \boldsymbol{i}_r \\ \boldsymbol{\phi}_r = L_{l,r} \cdot \boldsymbol{i}_r + \boldsymbol{L}_{r,r} \cdot \boldsymbol{i}_r + \boldsymbol{L}_{s,r}^{\mathrm{T}}(\theta_e) \cdot \boldsymbol{i}_s \end{cases}$$

Mechanical Equations

$$\begin{cases} \dfrac{\mathrm{d}\theta_m}{\mathrm{d}t} = \omega_m \\ J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{d}t} + F(\omega_m) \cdot \omega_m = T_{em} - T_m \\ T_{em} = P_p \cdot \boldsymbol{i}_s^{\mathrm{T}} \cdot \dfrac{\partial \boldsymbol{L}_{s,r}}{\partial \theta_e} \cdot \boldsymbol{i}_r \end{cases}$$

Electrical Angle Definition

$$\theta_e = P_p \theta_m \qquad \omega_e = P_p \omega_m$$

Inductances

$$\begin{cases} L_{m,s} = \dfrac{{N'_s}^2}{R_{m,eq}} \qquad L_{m,r} = \dfrac{{N'_r}^2}{R_{m,eq}} \qquad L_{m,sr} = \dfrac{N'_s N'_r}{R_{m,eq}} \\ \boldsymbol{L}_{s,s} = L_{m,s} \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix} \\ \boldsymbol{L}_{r,r} = L_{m,r} \begin{bmatrix} 1 & -1/2 & -1/2 \\ -1/2 & 1 & -1/2 \\ -1/2 & -1/2 & 1 \end{bmatrix} \\ \boldsymbol{L}_{s,r} = L_{m,sr} \begin{bmatrix} \cos(\theta_e) & \cos(\theta_e + 120°) & \cos(\theta_e + 240°) \\ \cos(\theta_e - 120°) & \cos(\theta_e) & \cos(\theta_e + 120°) \\ \cos(\theta_e - 240°) & \cos(\theta_e - 120°) & \cos(\theta_e) \end{bmatrix} \end{cases}$$

INDUCTION MACHINE SPACE VECTOR MODELING

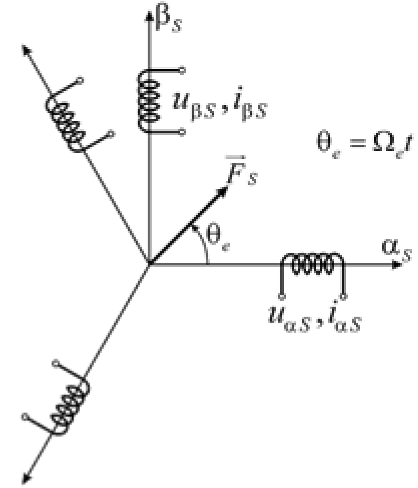
Modeling in the transformed domain

SPACE VECTOR FORMALISM

The mathematical model in the phase variable domain is normally considered too complex for practical purposes

Three variables have to be considered for the stator quantities, and three variables for the rotor quantities

There is no decoupling, even between stator-stator and rotor-rotor interactions

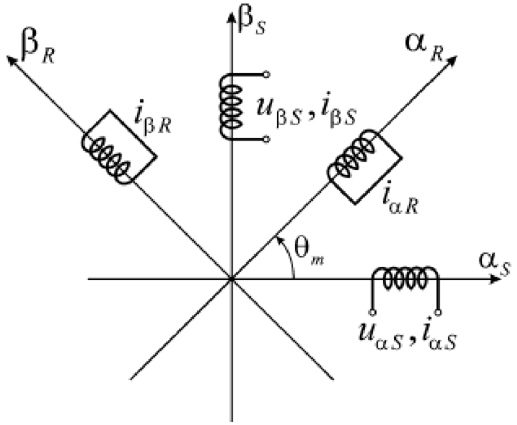


The mathematical model can be simplified by using the space vector formalism:

$$\begin{cases} \underline{x} = x_\alpha + j x_\beta = \frac{2}{3}\left(x_a + x_b\, e^{j2\pi/3} + x_c\, e^{j4\pi/3}\right) & \text{(Amplitude invariant definition)} \\ x_0 = \frac{1}{3}\left(x_a + x_b + x_c\right) & \text{(Zero Sequence component)} \end{cases}$$

$$\begin{bmatrix} x_\alpha \\ x_\beta \\ x_0 \end{bmatrix} = \frac{2}{3} \cdot \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \cdot \begin{bmatrix} x_a \\ x_b \\ x_c \end{bmatrix}$$

Matrix formulation
(Clarke's Transformation)



We can transform a three-phase set in an equivalent two-phase set

The zero-sequence components can be neglected in (almost) all cases

VOLTAGE BALANCE EQUATIONS

Phase variable domain (vector formalism)

$$\boldsymbol{v}_s = R_s \cdot \boldsymbol{i}_s + \frac{\mathrm{d}\boldsymbol{\phi}_s}{\mathrm{d}t} \qquad \boldsymbol{0} = \boldsymbol{v}_r = R_r \cdot \boldsymbol{i}_r + \frac{\mathrm{d}\boldsymbol{\phi}_r}{\mathrm{d}t}$$

Phase variable domain (expanded)

$$\begin{cases} v_{s,a} = R_s \cdot i_{s,a} + \dfrac{\mathrm{d}\phi_{s,a}}{\mathrm{d}t} \\ v_{s,b} = R_s \cdot i_{s,b} + \dfrac{\mathrm{d}\phi_{s,b}}{\mathrm{d}t} \\ v_{s,c} = R_s \cdot i_{s,c} + \dfrac{\mathrm{d}\phi_{s,c}}{\mathrm{d}t} \end{cases} \qquad \begin{cases} 0 = v_{r,a} = R_r \cdot i_{r,a} + \dfrac{\mathrm{d}\phi_{r,a}}{\mathrm{d}t} \\ 0 = v_{r,b} = R_r \cdot i_{r,b} + \dfrac{\mathrm{d}\phi_{r,b}}{\mathrm{d}t} \\ 0 = v_{r,c} = R_r \cdot i_{r,c} + \dfrac{\mathrm{d}\phi_{r,c}}{\mathrm{d}t} \end{cases}$$

Space Vector formalism (complex numbers)

$$\underline{v}_s = R_s \cdot \underline{i}_s + \frac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} \qquad 0 = \underline{v}_r = R_r \cdot \underline{i}_r + \frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t}$$

Space Vector formalism (stationary reference frame components)

$$\begin{cases} v_{s,\alpha} = R_s \cdot i_{s,\alpha} + \dfrac{\mathrm{d}\phi_{s,\alpha}}{\mathrm{d}t} \\ v_{s,\beta} = R_s \cdot i_{s,\beta} + \dfrac{\mathrm{d}\phi_{s,\beta}}{\mathrm{d}t} \end{cases} \qquad \begin{cases} 0 = v_{r,\alpha} = R_r \cdot i_{r,\alpha} + \dfrac{\mathrm{d}\phi_{r,\alpha}}{\mathrm{d}t} \\ 0 = v_{r,\beta} = R_r \cdot i_{r,\beta} + \dfrac{\mathrm{d}\phi_{r,\beta}}{\mathrm{d}t} \end{cases}$$

FLUXES EXPRESSIONS

Phase variable domain
(vector formalism)

$$\boldsymbol{\phi}_s = L_{l,s} \cdot \boldsymbol{i}_s + \boldsymbol{L}_{s,s} \cdot \boldsymbol{i}_s + \boldsymbol{L}_{s,r}(\theta_e) \cdot \boldsymbol{i}_r$$

$$\boldsymbol{\phi}_r = L_{l,r} \cdot \boldsymbol{i}_r + \boldsymbol{L}_{r,r} \cdot \boldsymbol{i}_r + \boldsymbol{L}_{s,r}^{\mathrm{T}}(\theta_e) \cdot \boldsymbol{i}_s$$

Space Vector formalism
(complex numbers)

$$\underline{\phi}_s = (L_{l,s} + 3/2 \cdot L_{m,s}) \cdot \underline{i}_s + (3/2 \cdot L_{m,sr}) \cdot \underline{i}_r \, \mathrm{e}^{\mathrm{j}\theta_e}$$
$$= L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r \, \mathrm{e}^{\mathrm{j}\theta_e}$$

$$\underline{\phi}_r = (L_{l,r} + 3/2 \cdot L_{m,r}) \cdot \underline{i}_r + (3/2 \cdot L_{m,sr}) \cdot \underline{i}_s \, \mathrm{e}^{-\mathrm{j}\theta_e}$$
$$= L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s \, \mathrm{e}^{-\mathrm{j}\theta_e}$$

Space Vector formalism
(stationary reference frame components)

$$\begin{bmatrix} \phi_{s,\alpha} \\ \phi_{s,\beta} \\ \phi_{r,\alpha} \\ \phi_{r,\beta} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m \cos(\theta_e) & -L_m \sin(\theta_e) \\ 0 & L_s & L_m \sin(\theta_e) & L_m \cos(\theta_e) \\ L_m \cos(\theta_e) & L_m \sin(\theta_e) & L_r & 0 \\ -L_m \sin(\theta_e) & L_m \cos(\theta_e) & 0 & L_r \end{bmatrix} \cdot \begin{bmatrix} i_{s,\alpha} \\ i_{s,\beta} \\ i_{r,\alpha} \\ i_{r,\beta} \end{bmatrix}$$

ELECTROMAGNETIC TORQUE

Phase variable domain
(vector formalism)

$$T_{em} = P_p \cdot \boldsymbol{i}_s^{\mathrm{T}} \cdot \frac{\partial \boldsymbol{L}_{s,r}}{\partial \theta_e} \cdot \boldsymbol{i}_r$$

⇩

Space Vector formalism
(complex numbers)

$$T_{em} = \frac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s \cdot \hat{\underline{i}}_r \cdot \mathrm{e}^{-\mathrm{j}\theta_e} \right\}$$

(hat used to denote the complex conjugate)

⇩

Space Vector formalism
(stationary reference frame components)

$$T_{em} = \frac{3}{2} P_p \cdot \begin{bmatrix} i_{s,\alpha} & i_{s,\beta} \end{bmatrix} \cdot \frac{\partial}{\partial \theta_e} \begin{bmatrix} L_m \cos(\theta_e) & -L_m \sin(\theta_e) \\ L_m \sin(\theta_e) & L_m \cos(\theta_e) \end{bmatrix} \cdot \begin{bmatrix} i_{r,\alpha} \\ i_{r,\beta} \end{bmatrix}$$

$$= \frac{3}{2} P_p \, L_m \cdot \left[(i_{r,\alpha} \, i_{s,\beta} - i_{r,\beta} \, i_{s,\alpha}) \cos(\theta_e) - (i_{r,\alpha} \, i_{s,\alpha} + i_{r,\beta} \, i_{s,\beta}) \sin(\theta_e) \right]$$

ELECTROMAGNETIC TORQUE – EQUIVALENT EXPRESSIONS

Based on the space vector formalism, other equivalent expressions of the torque can be obtained

They highlight different kind of interactions that exist between currents and fluxes

$$T_{em} = \frac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s \cdot \hat{\underline{i}}_r \cdot \mathrm{e}^{-\mathrm{j}\theta_e} \right\}$$ Stator Currents – Rotor Currents

$$= \frac{3}{2} P_p \, \mathrm{Im}\left\{ \underline{i}_s \cdot \hat{\underline{\phi}}_s \right\}$$ Stator Currents – Stator Fluxes

$$= \frac{3}{2} P_p \, \mathrm{Im}\left\{ \underline{\phi}_r \cdot \hat{\underline{i}}_r \right\}$$ Rotor Currents – Rotor Fluxes

$$= \frac{3}{2} P_p \, \frac{L_m}{L_s} \, \mathrm{Im}\left\{ \underline{\phi}_s \cdot \hat{\underline{i}}_r \cdot \mathrm{e}^{-\mathrm{j}\theta_e} \right\}$$ Stator Fluxes – Rotor Currents

$$= \frac{3}{2} P_p \, \frac{L_m}{L_s L_r - L_m^2} \, \mathrm{Im}\left\{ \underline{\phi}_s \cdot \hat{\underline{\phi}}_r \cdot \mathrm{e}^{-\mathrm{j}\theta_e} \right\}$$ Stator Fluxes – Rotor Fluxes

According to the chosen state variables for control, different formulations can be more convenient

INDUCTION MACHINE MODEL – SPACE VECTOR FORMALISM

Electrical Equations

$$\begin{cases} \underline{v}_s = R_s \cdot \underline{i}_s + \dfrac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} \\ 0 = \underline{v}_r = R_r \cdot \underline{i}_r + \dfrac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} \\ \underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r \, \mathrm{e}^{\mathrm{j}\theta_e} \\ \underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s \, \mathrm{e}^{-\mathrm{j}\theta_e} \end{cases}$$

Mechanical Equations

$$\begin{cases} \dfrac{\mathrm{d}\theta_m}{\mathrm{d}t} = \omega_m \\ J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{d}t} + F(\omega_m) \cdot \omega_m = T_{em} - T_m \\ T_{em} = \dfrac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s \cdot \hat{\underline{i}}_r \cdot \mathrm{e}^{-\mathrm{j}\theta_e} \right\} \end{cases}$$

Electrical Angle Definition

$$\theta_e = P_p \theta_m$$
$$\omega_e = P_p \omega_m$$

Inductances

$$\begin{cases} L_s = L_{l,s} + 3/2 \, L_{m,s} \\ L_r = L_{l,r} + 3/2 \, L_{m,r} \\ L_m = 3/2 \, L_{m,sr} \end{cases}$$

- The stator vectors are represented in the stator reference frame
- The roror vectors are represented in the rotor reference frame
- The two reference frames are moving with respect to one another

INDUCTION MACHINE
COMMON REFERENCE FRAME MODELING

Modeling in a common reference frame

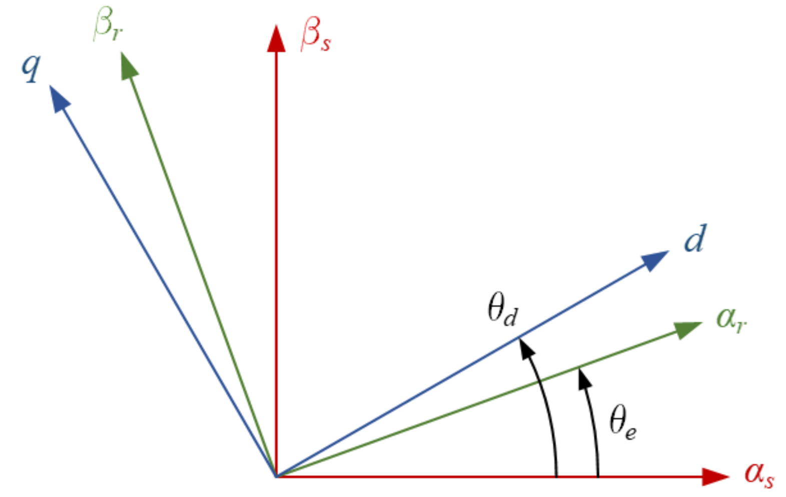
ROTATIONAL TRANSFORMATION

The mathematical model in the space vector formalism uses two distinct reference frames for stator and rotor vectors

The two reference frames are in relative motion with one another

It may be convenient to reformulate the model in a **common reference frame**

This can be done using the rotational transformation



$$\underline{x}^{\langle dq \rangle} = x_d + j x_q = \underline{x}^{\langle \alpha\beta \rangle}\, \mathrm{e}^{-\mathrm{j}\vartheta}$$

Complex formulation

$$\begin{bmatrix} x_d \\ x_q \end{bmatrix} = \begin{bmatrix} \cos(\vartheta) & \sin(\vartheta) \\ -\sin(\vartheta) & \cos(\vartheta) \end{bmatrix} \cdot \begin{bmatrix} x_\alpha \\ x_\beta \end{bmatrix}$$

Matrix formulation

The transformation angle depends on the initial reference frame and on the final reference frame

To transform the **Stator Variables**

$$\vartheta = \theta_d$$

To transform the **Rotor Variables**

$$\vartheta = \theta_d - \theta_e$$

(Slip angle)

The common reference frame **can be chosen arbitrarily**

The common reference frame can be a rotating reference frame, so that the steady state variables are constant

Angular speed of the common reference frame

$$\omega_d = \mathrm{d}\theta_d / \mathrm{d}t$$

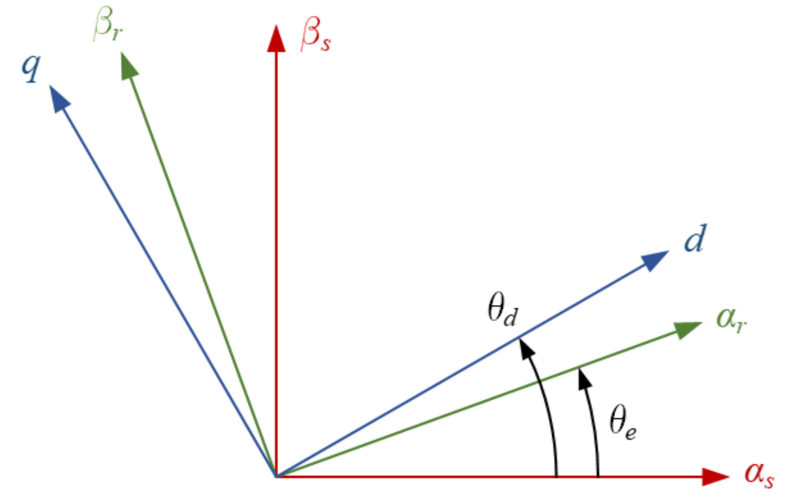
ROTATIONAL TRANSFORMATION – STATOR VOLTAGE BALANCE EQUATION

$$\underline{v}_s = R_s \cdot \underline{i}_s + \frac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t}$$

⇩

$$\underline{v}_s^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}\theta_d} = R_s \cdot \underline{i}_s^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}\theta_d} + \frac{\mathrm{d}(\underline{\phi}_s^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}\theta_d})}{\mathrm{d}t}$$

$$= R_s \cdot \underline{i}_s^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}\theta_d} + \frac{\mathrm{d}\underline{\phi}_s^{\langle dq \rangle}}{\mathrm{d}t} \mathrm{e}^{\mathrm{j}\theta_d} + \mathrm{j}\omega_d \underline{\phi}_s^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}\theta_d}$$



⇩

$$\underline{v}_s^{\langle dq \rangle} = R_s \cdot \underline{i}_s^{\langle dq \rangle} + \frac{\mathrm{d}\underline{\phi}_s^{\langle dq \rangle}}{\mathrm{d}t} + \mathrm{j}\omega_d \underline{\phi}_s^{\langle dq \rangle} \quad \Rightarrow \quad \begin{cases} v_{s,d} = R_s \cdot i_{s,d} + \dfrac{\mathrm{d}\phi_{s,d}}{\mathrm{d}t} - \omega_d \phi_{s,q} \\ v_{s,q} = R_s \cdot i_{s,q} + \dfrac{\mathrm{d}\phi_{s,q}}{\mathrm{d}t} + \omega_d \phi_{s,d} \end{cases}$$

► A **virtual motional-induced back-EMF** is generated by the reference frame transformation

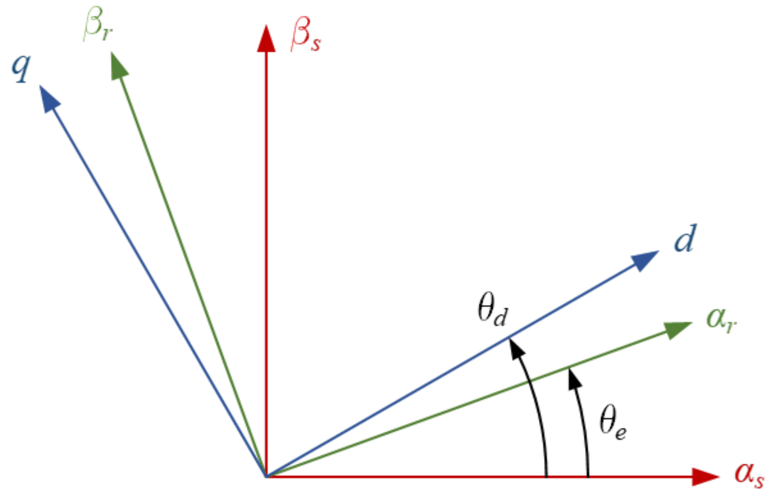
ROTATIONAL TRANSFORMATION – ROTOR VOLTAGE BALANCE EQUATION

$$0 = \underline{v}_r = R_r \cdot \underline{i}_r + \frac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t}$$

⇓

$$0 = \underline{v}_r^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)} = R_r \cdot \underline{i}_r^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)} + \frac{\mathrm{d}(\underline{\phi}_r^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)})}{\mathrm{d}t}$$

$$= R_r \cdot \underline{i}_r^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)} + \frac{\mathrm{d}\underline{\phi}_r^{\langle dq \rangle}}{\mathrm{d}t} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)}$$



⇓

$$0 = \underline{v}_r^{\langle dq \rangle} = R_r \cdot \underline{i}_r^{\langle dq \rangle} + \frac{\mathrm{d}\underline{\phi}_r^{\langle dq \rangle}}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r^{\langle dq \rangle}$$

⇒

$$\begin{cases} 0 = v_{r,d} = R_r \cdot i_{r,d} + \dfrac{\mathrm{d}\phi_{r,d}}{\mathrm{d}t} - (\omega_d - \omega_e)\phi_{r,q} \\ 0 = v_{r,q} = R_r \cdot i_{r,q} + \dfrac{\mathrm{d}\phi_{r,q}}{\mathrm{d}t} + (\omega_d - \omega_e)\phi_{r,d} \end{cases}$$

- A **virtual motional-induced back-EMF** is generated by the reference frame transformation

ROTATIONAL TRANSFORMATION – FLUXES

$$\underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r\, \mathrm{e}^{\mathrm{j}\theta_e}$$

$$\underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s\, \mathrm{e}^{-\mathrm{j}\theta_e}$$

⇓

$$\underline{\phi}_s^{\langle dq\rangle} \mathrm{e}^{\mathrm{j}\theta_d} = L_s \cdot \underline{i}_s^{\langle dq\rangle} \mathrm{e}^{\mathrm{j}\theta_d} + L_m \cdot \underline{i}_r^{\langle dq\rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)} \mathrm{e}^{\mathrm{j}\theta_e}$$

$$\underline{\phi}_r^{\langle dq\rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)} = L_r \cdot \underline{i}_r^{\langle dq\rangle} \mathrm{e}^{\mathrm{j}(\theta_d - \theta_e)} + L_m \cdot \underline{i}_s^{\langle dq\rangle} \mathrm{e}^{\mathrm{j}\theta_d} \mathrm{e}^{-\mathrm{j}\theta_e}$$

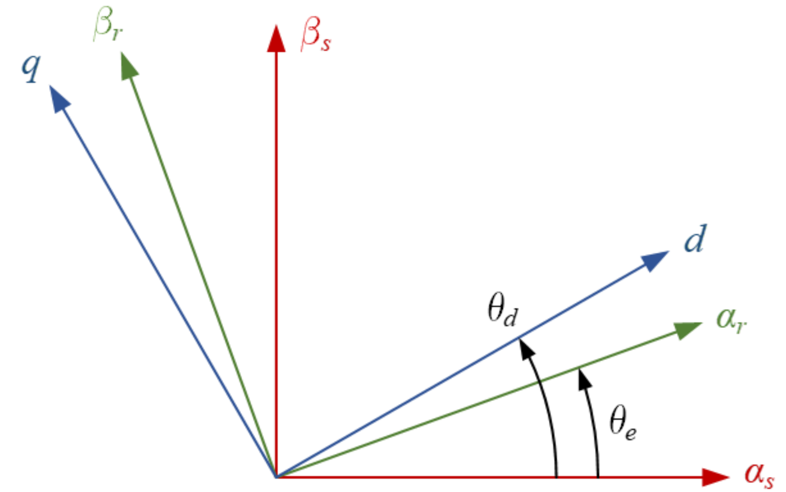
⇓

$$\underline{\phi}_s^{\langle dq\rangle} = L_s \cdot \underline{i}_s^{\langle dq\rangle} + L_m \cdot \underline{i}_r^{\langle dq\rangle}$$

$$\underline{\phi}_r^{\langle dq\rangle} = L_r \cdot \underline{i}_r^{\langle dq\rangle} + L_m \cdot \underline{i}_s^{\langle dq\rangle}$$

⇒

$$\begin{bmatrix} \phi_{s,d} \\ \phi_{s,q} \\ \phi_{r,d} \\ \phi_{r,q} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L_r & 0 \\ 0 & L_m & 0 & L_r \end{bmatrix} \cdot \begin{bmatrix} i_{s,d} \\ i_{s,q} \\ i_{r,d} \\ i_{r,q} \end{bmatrix}$$



- In the common reference frame, **the inductance matrix coefficients are all constant**

ROTATIONAL TRANSFORMATION – ELECTROMAGNETIC TORQUE

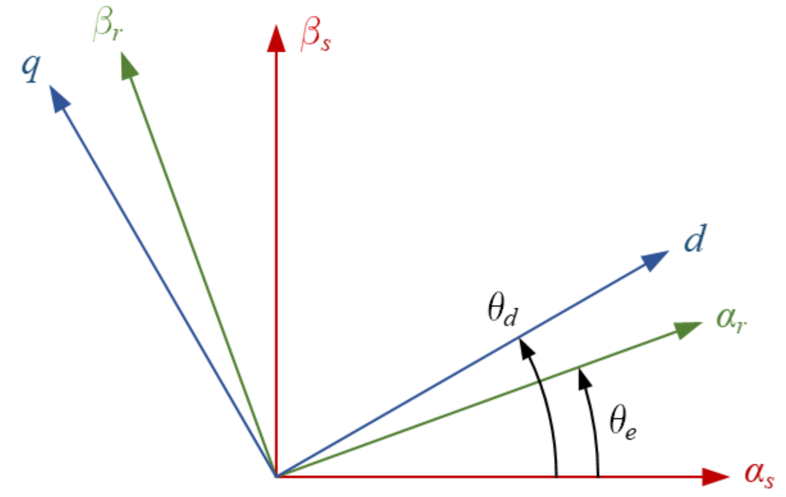
$$T_{em} = \frac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s \cdot \hat{\underline{i}}_r \cdot \mathrm{e}^{-\mathrm{j}\theta_e} \right\}$$

⇓

$$T_{em} = \frac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s^{\langle dq \rangle} \mathrm{e}^{\mathrm{j}\theta_d} \cdot \hat{\underline{i}}_r^{\langle dq \rangle} \mathrm{e}^{-\mathrm{j}(\theta_d - \theta_e)} \cdot \mathrm{e}^{-\mathrm{j}\theta_e} \right\}$$

$$= \frac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s^{\langle dq \rangle} \cdot \hat{\underline{i}}_r^{\langle dq \rangle} \right\}$$

$$= \frac{3}{2} P_p \, L_m \left(i_{sq} \, i_{rd} - i_{sd} \, i_{rq} \right)$$



- In the common reference frame, **the torque expression does not depend on the rotor position**

ELECTROMAGNETIC TORQUE – EQUIVALENT EXPRESSIONS

In the common reference frame, the equivalent expressions of the electromagnetic torque are independent from the rotor angle

A non-zero average torque can be developed at any rotor speed, as long as the electrical variables are synchronous

$$T_{em} = \frac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s^{\langle dq \rangle} \cdot \hat{\underline{i}}_r^{\langle dq \rangle} \right\}$$ Stator Currents – Rotor Currents

$$= \frac{3}{2} P_p \, \mathrm{Im}\left\{ \underline{i}_s^{\langle dq \rangle} \cdot \hat{\underline{\phi}}_s^{\langle dq \rangle} \right\}$$ Stator Currents – Stator Fluxes

$$= \frac{3}{2} P_p \, \mathrm{Im}\left\{ \underline{\phi}_r^{\langle dq \rangle} \cdot \hat{\underline{i}}_r^{\langle dq \rangle} \right\}$$ Rotor Currents – Rotor Fluxes

$$= \frac{3}{2} P_p \, \frac{L_m}{L_s} \, \mathrm{Im}\left\{ \underline{\phi}_s^{\langle dq \rangle} \cdot \hat{\underline{i}}_r^{\langle dq \rangle} \right\}$$ Stator Fluxes – Rotor Currents

$$= \frac{3}{2} P_p \, \frac{L_m}{L_s L_r - L_m^2} \, \mathrm{Im}\left\{ \underline{\phi}_s^{\langle dq \rangle} \cdot \hat{\underline{\phi}}_r^{\langle dq \rangle} \right\}$$ Stator Fluxes – Rotor Fluxes

According to the chosen state variables for control, different formulations can be more convenient

INDUCTION MACHINE MODEL – MODEL IN A COMMON REFERENCE FRAME

Electrical Equations

$$\begin{cases} \underline{v}_s^{\langle dq \rangle} = R_s \cdot \underline{i}_s^{\langle dq \rangle} + \dfrac{\mathrm{d}\underline{\phi}_s^{\langle dq \rangle}}{\mathrm{d}t} + \mathrm{j}\omega_d \underline{\phi}_s^{\langle dq \rangle} \\ 0 = \underline{v}_r^{\langle dq \rangle} = R_r \cdot \underline{i}_r^{\langle dq \rangle} + \dfrac{\mathrm{d}\underline{\phi}_r^{\langle dq \rangle}}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e) \underline{\phi}_r^{\langle dq \rangle} \\ \underline{\phi}_s^{\langle dq \rangle} = L_s \cdot \underline{i}_s^{\langle dq \rangle} + L_m \cdot \underline{i}_r^{\langle dq \rangle} \\ \underline{\phi}_r^{\langle dq \rangle} = L_r \cdot \underline{i}_r^{\langle dq \rangle} + L_m \cdot \underline{i}_s^{\langle dq \rangle} \end{cases}$$

Electrical Angle Definition

$$\theta_e = P_p \theta_m$$

Mechanical Equations

$$\begin{cases} \dfrac{\mathrm{d}\theta_m}{\mathrm{d}t} = \omega_m \\ J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{d}t} + F(\omega_m) \cdot \omega_m = T_{em} - T_m \\ T_{em} = \dfrac{3}{2} P_p \, L_m \, \mathrm{Im}\left\{ \underline{i}_s^{\langle dq \rangle} \cdot \hat{\underline{i}}_r^{\langle dq \rangle} \right\} \end{cases}$$

The common reference frame **can be chosen arbitrarily**

There are some preferred choices for reference frames of interest:

- Stationary (Fixed) reference frame $\theta_d = 0$ $\quad \omega_d = 0$
- Rotor oriented reference frame $\theta_d = \theta_m$ $\quad \omega_d = \omega_m$
- Stator Flux oriented reference frame $\theta_d = \angle \underline{\phi}_s$
- Rotor Flux oriented reference frame $\theta_d = \angle \underline{\phi}_r$

INDUCTION MACHINE MODEL – MODEL IN A COMMON REFERENCE FRAME

Electrical Equations

$$\begin{cases} \underline{v}_s = R_s \cdot \underline{i}_s + \dfrac{\mathrm{d}\underline{\phi}_s}{\mathrm{d}t} + \mathrm{j}\omega_d \underline{\phi}_s \\ 0 = \underline{v}_r = R_r \cdot \underline{i}_r + \dfrac{\mathrm{d}\underline{\phi}_r}{\mathrm{d}t} + \mathrm{j}(\omega_d - \omega_e)\underline{\phi}_r \\ \underline{\phi}_s = L_s \cdot \underline{i}_s + L_m \cdot \underline{i}_r \\ \underline{\phi}_r = L_r \cdot \underline{i}_r + L_m \cdot \underline{i}_s \end{cases}$$

Mechanical Equations

$$\begin{cases} \dfrac{\mathrm{d}\theta_m}{\mathrm{d}t} = \omega_m \\ J \cdot \dfrac{\mathrm{d}\omega_m}{\mathrm{d}t} + F(\omega_m) \cdot \omega_m = T_{em} - T_m \\ T_{em} = \dfrac{3}{2} P_p L_m \,\mathrm{Im}\left\{\underline{i}_s \cdot \hat{\underline{i}}_r\right\} \end{cases}$$

Electrical Angle Definition

$$\theta_e = P_p \theta_m$$
$$\omega_e = P_p \omega_m$$

- Note: the rotor angle does not appear explicitly in the model

The subscripts <dq> can be removed to simplify the notation

The common reference frame **can be chosen arbitrarily**

There are some preferred choices for reference frames of interest:

- Stationary (Fixed) reference frame $\theta_d = 0$ $\omega_d = 0$
- Rotor oriented reference frame $\theta_d = \theta_m$ $\omega_d = \omega_m$
- Stator Flux oriented reference frame $\theta_d = \angle \underline{\phi}_s$
- Rotor Flux oriented reference frame $\theta_d = \angle \underline{\phi}_r$

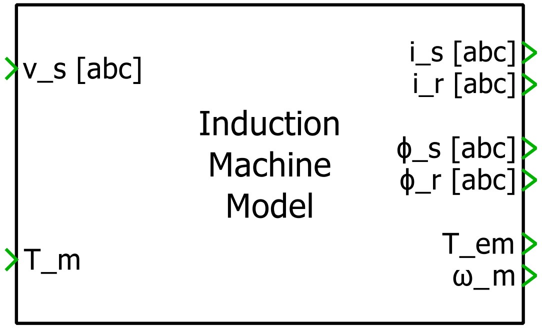
SUMMARY

Mathematical model of the Induction machine

BLOCK DIAGRAM OF THE MODEL

With the help of Laplace transform block diagram of model is obtained

- The inputs into model are: stator voltages (controlled inputs) and loading torque (disturbance input)
- The state variables can be chosen as stator currents, stator fluxes, rotor currents and rotor fluxes
- The outputs are: stator currents, rotor currents, stator fluxes, rotor fluxes, electromagnetic torque and rotor speed



(To do in the Exercise Session)